Field theory using xAct

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Gravity Workshop 2020

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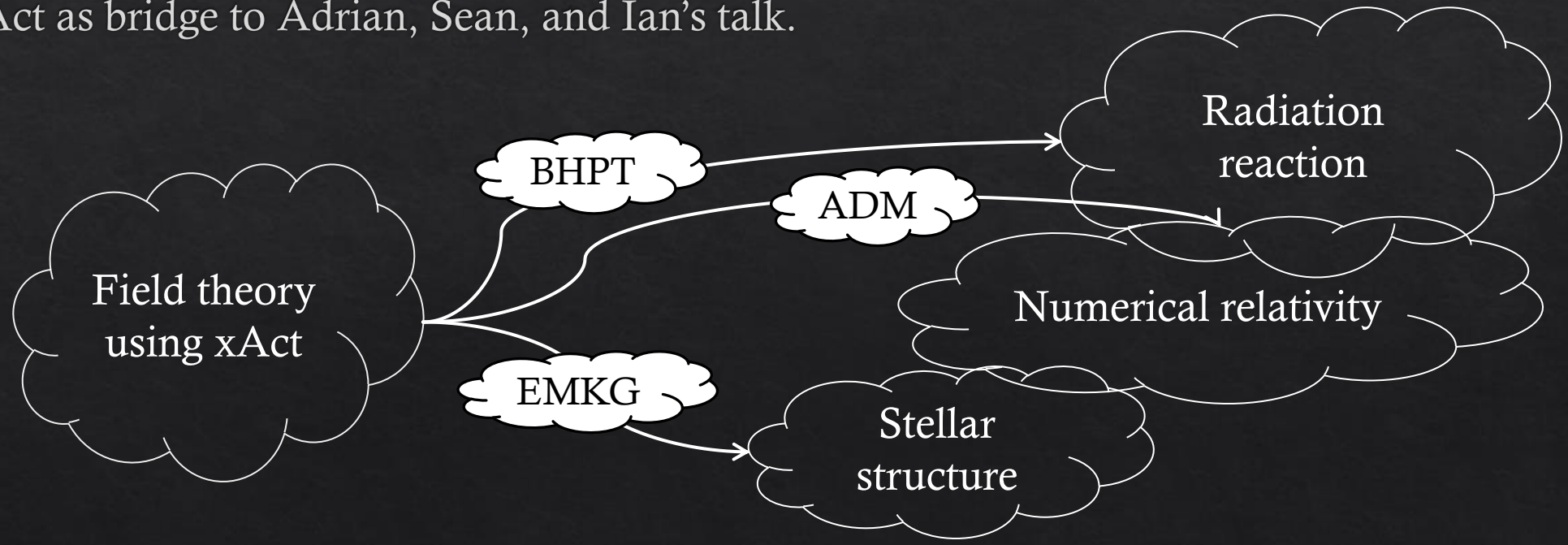
Objective

- Introduce field theory using xAct;
- Discuss Einstein-Maxwell-Klein-Gordon theory, black hole perturbation theory, and Arnowitt-Deser-Misner formalism;
- Act as bridge to Adrian, Sean, and Ian's talk.



Field theory

- *Field theory*: Action -> Field equations -> Solutions (BH, Stars, etc.)

Action | *Field equations*

$$S[\Phi] = \int d^4x \left(\frac{1}{8\pi G} \nabla\Phi(t,\vec{x}) \cdot \nabla\Phi(t,\vec{x}) + \rho(t,\vec{x})\,\Phi(t,\vec{x}) \right) \quad \Rightarrow \quad \nabla^2\Phi(t,\vec{x}) = 4\pi G \rho(t,\vec{x})$$

$$S[g_{ab}] = \int d^4x \sqrt{-g} \left(\frac{c^4}{16\pi G} R \right) + S_M[\Psi, g_{ab}] \quad \Rightarrow \quad G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

*Variational calculus/*xAct

1. Ch. 4 of [Carroll, S. M. (2019). *Spacetime and geometry*. Cambridge University Press.]
2. Final Ch. of [Goldstein, H., Poole, C., & Safko, J. (2002). *Classical mechanics*.]

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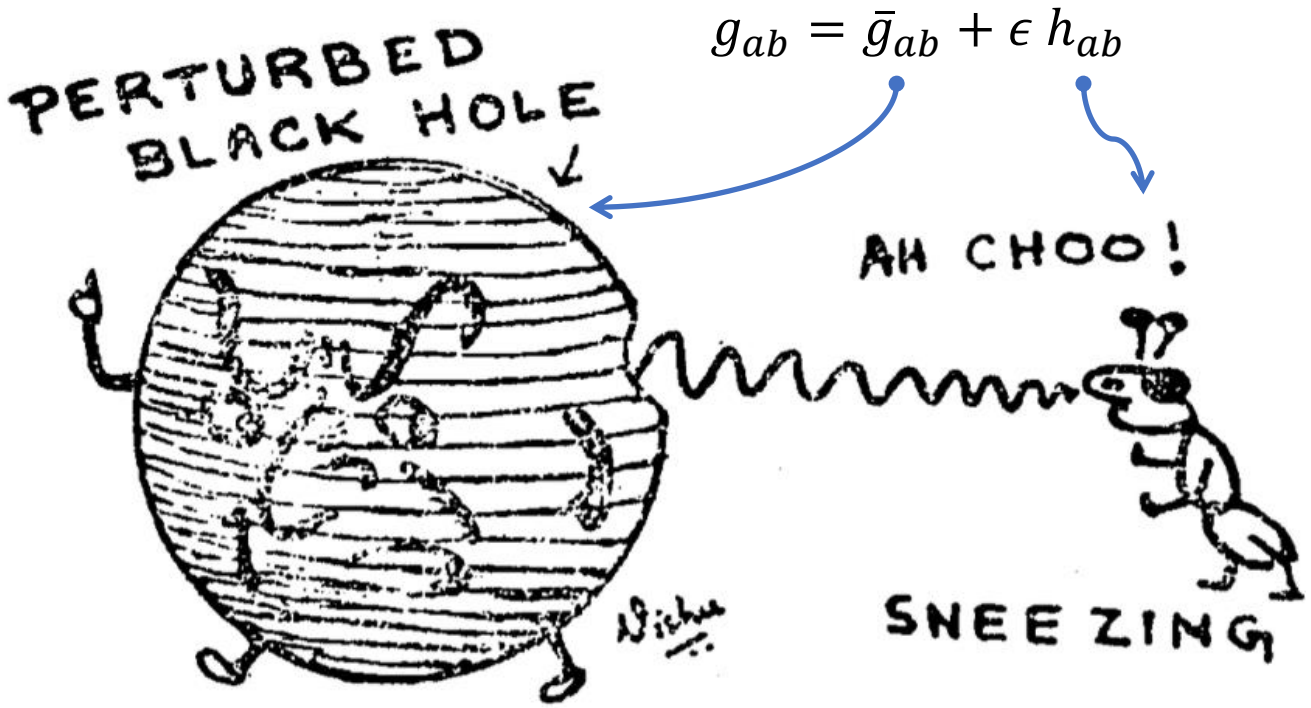
Black hole perturbation theory



Black hole? *Kerr-Newman*

(Nonrotating) Black hole + Ant?

- *Regge-Wheeler* equation
- *Zerilli* equation

$$-\partial_x^2\psi + V(x)\psi = \omega^2\psi$$

$$\psi = \text{master function} \sim |h_{ab}|$$

Black hole and sneezing ant,
<https://thewire.in/science/vishveshwara-quasi-normal-blackhole>

3. Regge, T., & Wheeler, J. A. (1957). *PR*, *108*(4), 1063.
4. Zerilli, F. J. (1970). *PRL*, *24*(13), 737.

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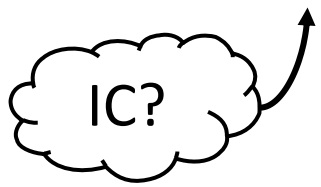
ADM formalism

g_{ab} = spacetime -> *all space and time; not how we do physics!*

Analogy:

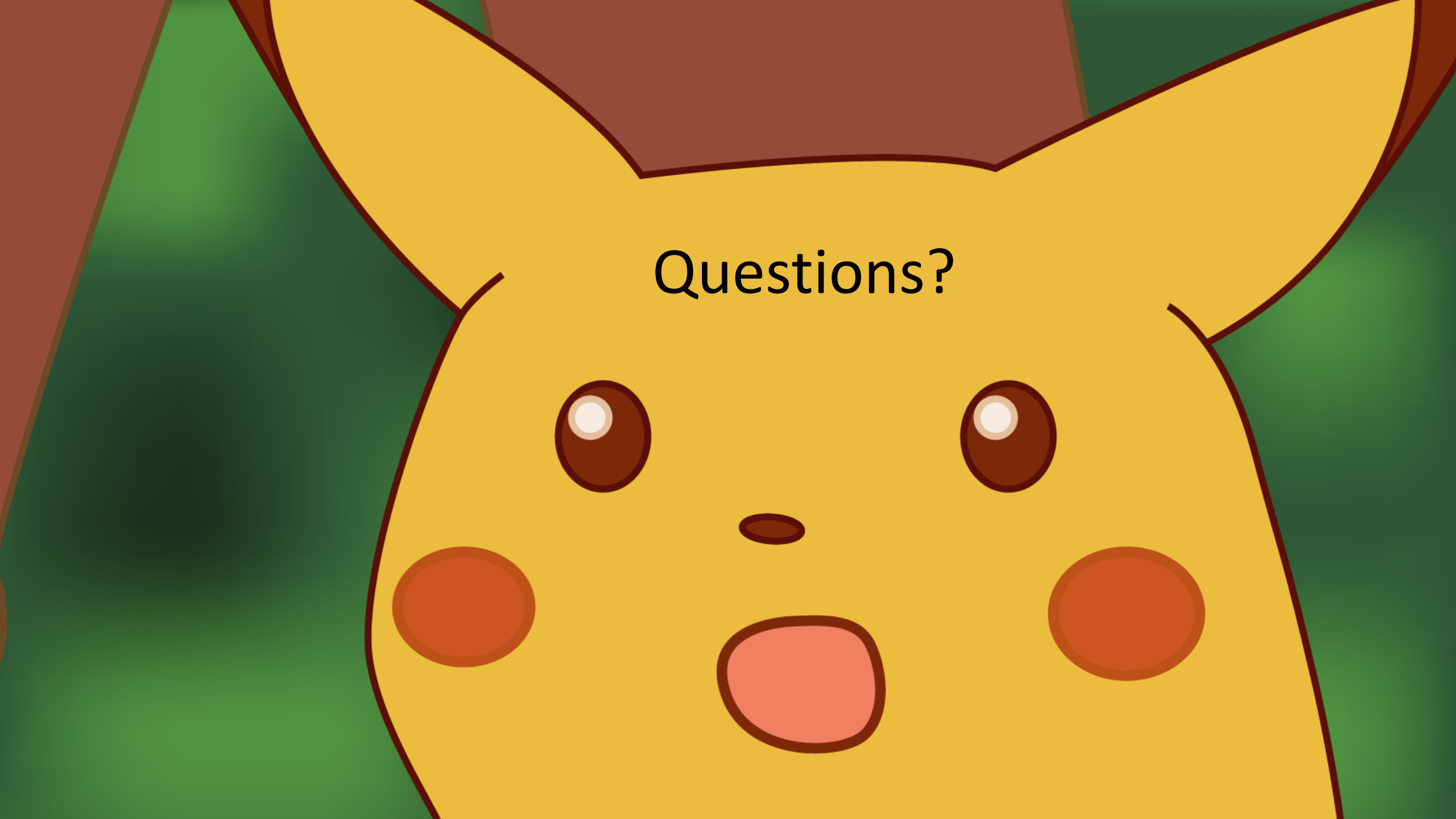
$$g_{ab} = \{\dots, 2000, 2001, \dots, 2019, 2020, \dots\}$$

$$\text{HWDP} = \cdots \mathbf{2000} \rightarrow\ 2001 \rightarrow \cdots \rightarrow 2019 \rightarrow 2020 \dots$$



Initial value formulation of GR -> *ADM formalism* -> Numerical relativity

Poisson, E. (2004). *A relativist's toolkit: the mathematics of black-hole mechanics*. Cambridge university press.


Questions?