

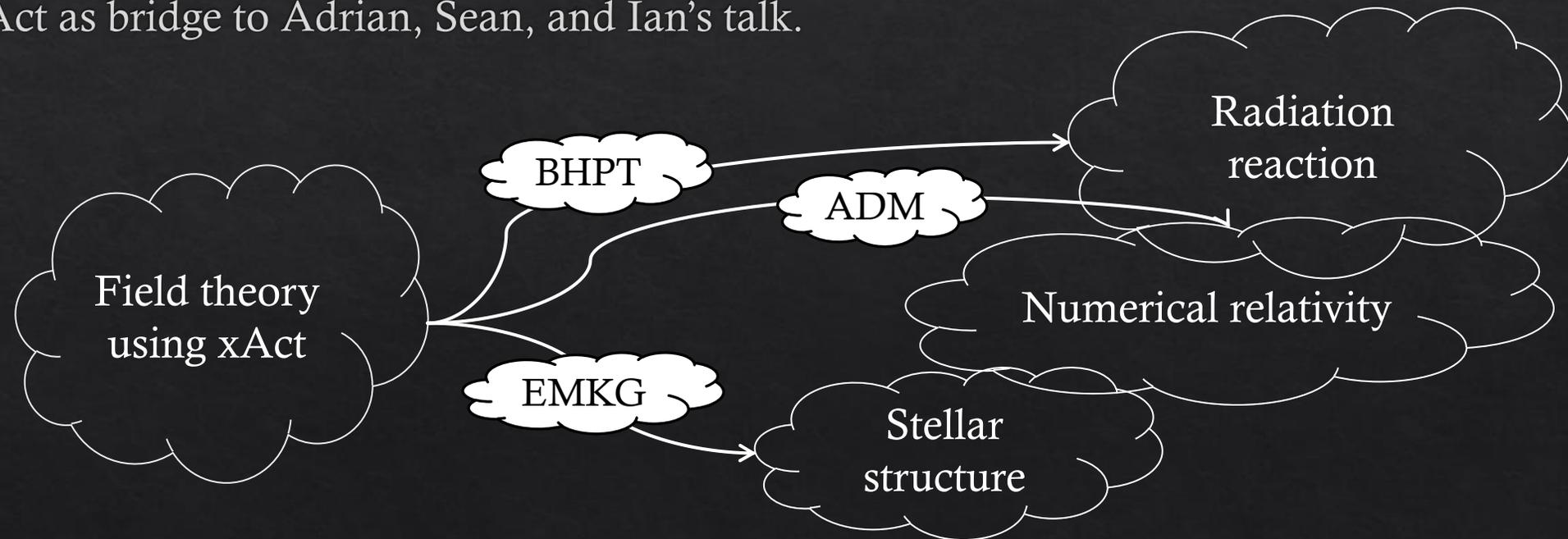
# Field theory using xAct

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*Gravity Workshop 2020*

# Objective

- ◆ Introduce field theory using xAct;
- ◆ Discuss Einstein-Maxwell-Klein-Gordon theory, black hole perturbation theory, and Arnowitt-Deser-Misner formalism;
- ◆ Act as bridge to Adrian, Sean, and Ian's talk.



# Field theory

- *Field theory: Action* -> Field equations -> Solutions (BH, Stars, etc.)

*Action*

$$S[\Phi] = \int d^4x \left( \frac{1}{8\pi G} \nabla\Phi(t, \vec{x}) \cdot \nabla\Phi(t, \vec{x}) + \rho(t, \vec{x}) \Phi(t, \vec{x}) \right)$$

$$S[g_{ab}] = \int d^4x \sqrt{-g} \left( \frac{c^4}{16\pi G} R \right) + S_M[\Psi, g_{ab}]$$

*Field equations*

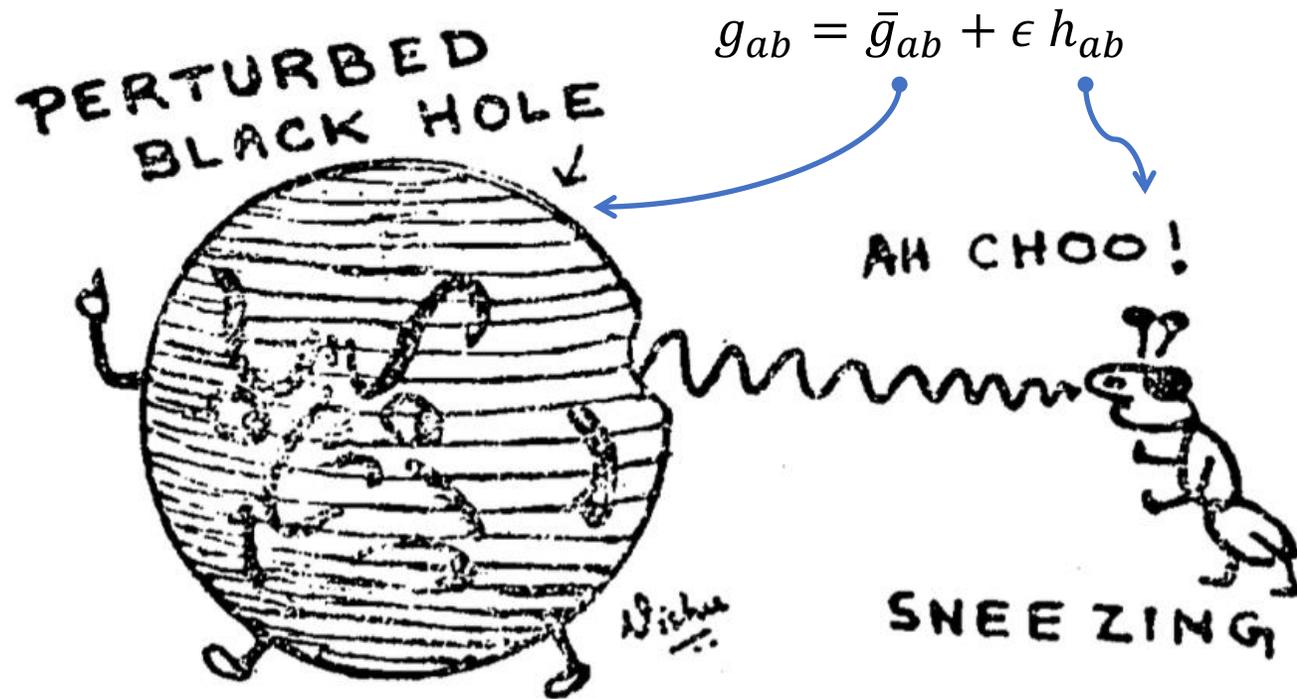
$$\nabla^2\Phi(t, \vec{x}) = 4\pi G\rho(t, \vec{x})$$

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

*Variational calculus/xAct*

1. Ch. 4 of [Carroll, S. M. (2019). *Spacetime and geometry*. Cambridge University Press.]
2. Final Ch. of [Goldstein, H., Poole, C., & Safko, J. (2002). *Classical mechanics*.]

# Black hole perturbation theory



Black hole? *Kerr-Newman*

(Nonrotating) Black hole + Ant?

- *Regge-Wheeler* equation
- *Zerilli* equation

$$-\partial_x^2 \psi + V(x)\psi = \omega^2 \psi$$

$\psi =$  master function  $\sim |h_{ab}|$

*Black hole and sneezing ant,*  
<<https://thewire.in/science/vishveshwara-quasi-normal-blackhole>>

3. Regge, T., & Wheeler, J. A. (1957). *PR*, 108(4), 1063.

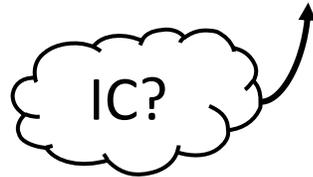
4. Zerilli, F. J. (1970). *PRL*, 24(13), 737.

# ADM formalism

$g_{ab}$  = spacetime  $\rightarrow$  *all space and time; not how we do physics!*

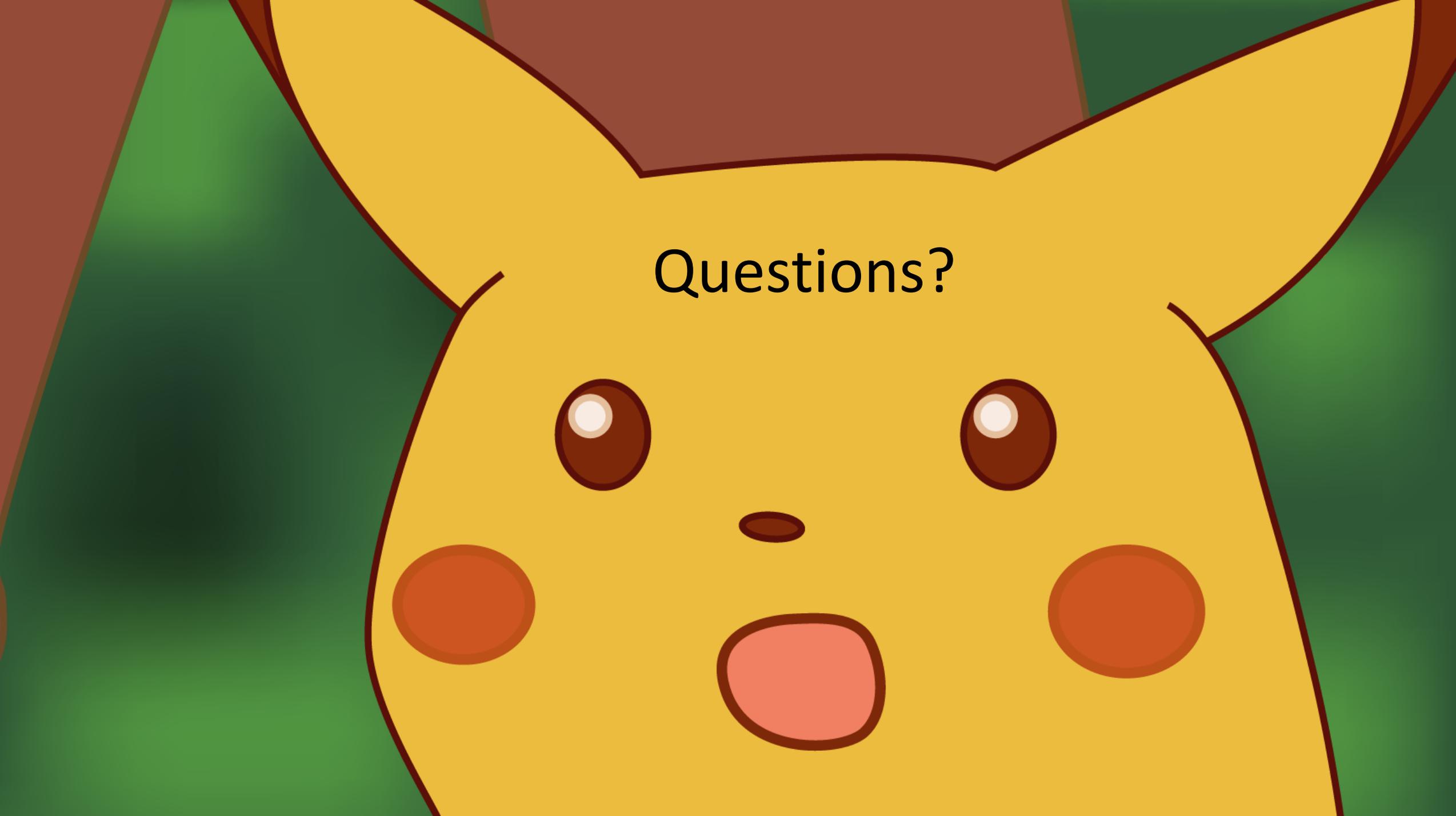
Analogy:

$g_{ab} = \{ \dots, 2000, 2001, \dots, 2019, 2020, \dots \}$   
HWDP =  $\dots$  **2000**  $\rightarrow$  2001  $\rightarrow$   $\dots$   $\rightarrow$  2019  $\rightarrow$  2020  $\dots$



Initial value formulation of GR  $\rightarrow$  *ADM formalism*  $\rightarrow$  Numerical relativity

Poisson, E. (2004). *A relativist's toolkit: the mathematics of black-hole mechanics*. Cambridge university press.

A stylized, cartoonish illustration of a yellow Pikachu's face. The face is centered and occupies most of the frame. It has large, round brown eyes with white highlights, a small brown nose, and a wide, open pink mouth. Two orange circular cheeks are visible on either side. The background consists of green and brown geometric shapes. The word "Questions?" is written in a simple black font above the eyes.

Questions?